Mid-term review

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Review

1. Let S be a subspace of $\mathbb{R}^n$ and let P_S be an orthogonal projection onto S. Show that $\dim(S) = Tr(P_S)$
2. True or False:
    1. Rank of a symmetric matrix is equal to the number of distinct eigenvalues
    2. Eigenvectors associated with distinct eigenvalues are orthogonal for a symmetric matrix
    3. Eigenvectors associated with the same eigenvalue cannot be orthogonal for a symmetric matrix
    4. Let $A, B \in \mathbb{R}^{n\times \mathrm{n}}$. If $v \in \mathbb{R}^n$ is an eigenvector of A and B, then v is an eigenvector of A+B
    5. Let $A, B \in \mathbb{R}^{n\times \mathrm{n}}$. If $v \in \mathbb{R}^n$ is an eigenvector of A and B, then v is an eigenvector of AB
    6. Let $A \in \mathbb{R}^{n\times \mathrm{n}}\ and\ v_1, v_2 \in \mathbb{R}^n$ be two eigenvectors of A associated with the eigenvalue λ. Any vector in the $span(v_1, v_2)$ is also an eigenvector of A associated with the eigenvalue λ
    7. If the input vectors $u_1, \dots, u_n$ are already orthogonal, the output of Gram Schmidt algorithm will be $u_1, \dots, u_n$

Review

1. Let x $\in \mathbb{R}^n \setminus \mathbf{0}$ $such\ that\ M = xx^T$
   1. Is M symmetric?
   2. What is the rank of M?
   3. What are the eigenvalues of M and their multiplicity?
2. Let $A \in \mathbb{R}^{n\times \mathrm{n}}$ be a symmetric matrix. Let $u_1, \dots, u_n$ be an orthonormal basis of $\mathbb{R}^n$ consisting of eigenvectors of A and let $\lambda_1, \dots, \lambda_n$ be the corresponding eigenvalues ($Au_i = \lambda_i u_i \; \forall \; i$). Give orthonormal basis of Ker(A) and Im(A) in terms of vectors $u_1, \dots u_n$