Mid-term review

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Review

1. Let $A, B \in \mathbb{R}^{m \times n}$. For each one of the subsets of $\mathbb{R}^n$ below say whether it is a subspace of $\mathbb{R}^n$ and justify your answer:
    1. $E_1 = \{x \in \mathbb{R}^n \mid Ax = 0\}$
    2. $E_2 = \{x \in \mathbb{R}^n \mid Ax = Bx\}$
    3. $E_3 = \{x \in \mathbb{R}^n \mid Ax = e_1\}$
    4. $E_4 = \{x \in \mathbb{R}^n \mid Ax \in span(e_1)\}$
    5. $E_5 = \{x \in \mathbb{R}^n \mid \sum_{i=0}^{n} (Ax)_i = 0\}$
2. Let $n > m$ and let $A \in \mathbb{R}^{n \times \mathrm{m}}$. Assume that A is full rank meaning that $rank(A) = \min(m, n) = m$.
    1. Does $Ax = b$ has a solution $\forall\ b \in \mathbb{R}^n$
    2. True or False: $\exists\ b \in \mathbb{R}^n$ for which there exists two distinct solutions $x \neq x'\, such\ that\ Ax = Ax' = b$

Review

1. Let $A \in \mathbb{R}^{m \times n}$
    1. Prove that $Ker(A^T)$ and $Im(A)$ are orthogonal to each other
    2. This raises the question: Is $Ker(A^T) = Im(A)^\perp$
2. True or False:
    1. There can exist a set of n non-zero orthogonal vectors in $\mathbb{R}^m$ if $n > m$
    2. The matrix corresponding to an orthogonal projection is symmetric
    3. The matrix corresponding to an orthogonal projection is orthogonal
    4. For any subspace S of $\mathbb{R}^n$, the orthogonal projection matrix onto S is unique
3. Let S be a subspace of $\mathbb{R}^n$ and let P_S be an orthogonal projection onto S. Show that $\dim(S) = Tr(P_S)$