Recitation - Week 6

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Stochastic matrices

1. Prove that the product of two stochastic matrices is a stochastic matrix
2. Let $A \in R^{n \times n}$ be a stochastic matrix. True or False:
    1. A is always invertible
    2. The eigenvector corresponding to the largest eigenvalue of A is unique
    3. A cannot have zero as its eigenvalue
3. Let $A \in \mathbb{R}^{n \times n}$ be a stochastic matrix. Prove that the absolute value of any eigenvalue of A is ≤ 1

Spectral decomposition

1. Let $A \in \mathbb{R}^{n \times n}$ be a symmetric matrix. Give a vector v with $||v|| = 1$ such that $||Av||$ is maximized
2. A symmetric matrix $M \in \mathbb{R}^{n \times n}$ is said to be positive definite if all of its eigenvalues are greater than zero. Prove that $x^T Mx > 0$ for a symmetric positive definite matrix $M \in \mathbb{R}^{n \times n}$ and $x \in \mathbb{R}^n$
3. Suppose S and T are symmetric and positive definite (all eigenvalues greater than zero)
    1. True or False: Product of two symmetric matrices will always be symmetric
    2. Prove that all eigenvalues of ST are still positive