Recitation - Week 5

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Eigenvalues and diagonalizable matrices

1. Let $A \in \mathbb{R}^{n\times n}$. Suppose that A has an eigenvalue λ with eigenvector $v \in \mathbb{R}^n$ associated to λ. Prove that
   i. $\forall\ \alpha \in \mathbb{R}, \lambda + \alpha$ is an eigenvalue of the matrix $A + \alpha I_{n\times \mathrm{n}}$ with corresponding eigenvector v
   ii. $\forall\ k \in \mathbb{N}, \lambda^k$ is an eigenvalue of the matrix A^k with corresponding eigenvector v
2. Let $A \in \mathbb{R}^{m\times n}$, $\mathrm{B} \in \mathbb{R}^{m\times m}$ and $C \in \mathbb{R}^{n\times n}$. Prove that if B and C are invertible, then rank(A)=rank(BAC). What does this say about rank of any diagonalizable matrix $X \in \mathbb{R}^{n\times n}$? Prove that $Tr(X) = \sum_i \lambda_i$ where λ_i are the eigenvalues of X.
3. Let $v_1, \dots, v_k$ be the eigenvectors of A corresponding respectively to eigenvalues $\lambda_1, \dots, \lambda_k$ such that all λ_i are distinct (assume that $\lambda_1 > \lambda_2 > \cdots > \lambda_k$). Prove the eigenvectors $v_1, \dots, v_k$ are linearly independent

Gram-Schmidt

1. Let $A \in \mathbb{R}^{m\times \mathrm{n}}$ have linearly independent columns. Show that there is a matrix $Q \in \mathbb{R}^{m\times \mathrm{n}}$ and $R \in \mathbb{R}^{n\times \mathrm{n}}$ such that A=QR, Q has orthonormal columns and R is upper triangular
2. Suppose $D \in \mathbb{R}^{n\times n}$ is diagonal. Give a vector $v \in \mathbb{R}^n$ with $||v|| = 1$ such that $||Dv||$ is maximized