Recitation - Week 4

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Projections

1. Suppose $v_1, .., v_m \in \mathbb{R}^k$ are m non-zero orthogonal vectors. Prove that they are linearly independent.
2. Let U be the subspace of $\mathbb{R}^n$ with orthonormal basis $u_1, \dots, u_k$.
   i. Prove that the projection of $x \in \mathbb{R}^n$ can be expressed as $P_U(x) =< u_1, x > u_1 +< u_2, x > u_2 + \dots + < u_k, x > u_k$ (Use the fact that the orthonormal basis for a subspace of $\mathbb{R}^n$ can be extended to obtain an orthonormal basis for $\mathbb{R}^n$)
   ii. Prove that $\left|\left|P_U(x)\right|\right| \leq \left|\left|x\right|\right|$
   iii. Prove that $x - P_U(x)$ is orthogonal to the subspace U
   iv. Show that the linear transformation $P_U : \mathbb{R}^n \to \mathbb{R}^n$ satisfies $P_U^2 = P_U \; and \; P_U^T = P_U$

Norm and inner product

1. Which of the following are an inner product for $x, y \in \mathbb{R}^3$:
   i. $f(x, y) = x_1 y_2 + x_2 y_3 + x_3 y_1$
   ii. $f(x, y) = x_1^2 y_1^2 + x_2^2 y_2^2 + x_3^2 y_3^2$
   iii. $f(x, y) = x_1 y_1 + x_2 y_2$
2. Let $x = (cos\theta_1, sin\theta_1) \in \mathbb{R}^2$ and $y = (cos\theta_2, sin\theta_2) \in \mathbb{R}^2$ be two vectors on unit circle ($||x|| = 1 = ||y||$). What does $x^T y$ represent geometrically?
3. For $x, y \in \mathbb{R}^n$, prove that $||x + y|| \leq ||x|| + ||y||$ (Triangular inequality). When does $||x + y|| = ||x|| + ||y||$?
4. For any $A \in \mathbb{R}^{m \times n}$ and $x \in \mathbb{R}^n$, show that

$$||Ax|| \leq ||x|| \sqrt{\sum_{i=1}^{m} \sum_{j=1}^{n} A_{ij}^2}$$