Recitation Week 10

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Convexity

1. For $f: \mathbb{R}^n \to \mathbb{R}$, define epigraph $epi(f) \subset \mathbb{R}^{n+1}$ to be set of all the points above the graph of f:

$$epi(f) = \{(x, t) \in \mathbb{R}^{n+1}: t \geq f(x)\}$$

   Prove that f is convex if and only if epi(f) is convex

2. Let $f: \mathbb{R}^n \to \mathbb{R}$ be given by $f(x) = x^T Ax$ for some symmetric matrix $A \in \mathbb{R}^{n \times n}$. Give conditions on A so that 0 is the global minimizer of f

Convexity

1. True or False:
    1. If f has only 1 global minima and no local minima, then f is convex
    2. Linear combination of two convex functions is convex
    3. Convex functions are differentiable at all points
    4. Norms are convex functions
    5. If f is convex, then $g(x) = f(Ax - b)$ is also convex in x
    6. Sum of a non-convex function (like $\cos(x)$) with another function can never be convex
    7. Union of convex sets is convex
    8. Intersection of convex sets is convex
    9. Maximum of two convex functions is convex
    10. If f is convex, f^n is also convex for $n \in \mathbb{N}$
    11. Every subspace is a convex set
    12. Every convex set is a subspace