Recitation Week 10

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Rank k approximation

1. Let $A \in \mathbb{R}^{m \times n}$ be a matrix such that using SVD, $A = U\Sigma V^T$. Let U_r and V_r denote the first r columns of U and V respectively

   i. Let $A \in \mathbb{R}^{m \times n}$ be a rank one matrix. Find the value of the square of the its first singular value in terms of entries of A

   ii. Let's say we have a function f to measure distance between two matrices such that

   $$f: X, Y \to \mathbb{R}; \quad \mathrm{X, Y} \in \mathbb{R}^{m \times n}$$

   How would you formulate the optimization problem to find a rank k matrix A_k which is at minimal distance to A?

SVD

1. Let $A \in \mathbb{R}^{m \times n}$ be a rectangular matrix such that using SVD, $A = U\Sigma V^T$ and let $rank(A) = r$. Let U_r and V_r denote the first r columns of U and V respectively
    1. Show that A can also be expressed as $A = XY^T$ where $X \in \mathbb{R}^{m \times r}$ and $\mathrm{Y} \in \mathbb{R}^{n \times r}$ such that X and Y have orthogonal columns respectively. Is this factorization unique?
    2. Computationally, what's the advantage of expressing $A \in \mathbb{R}^{m \times n}$ as $\mathrm{A} = XY^T$