Concept check - Week 2

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Concept Check

1. Suppose U is a subspace of V with $U \neq V$. Let $S: U \to W$ be a linear transformation and $S \neq 0$ (which means that $Su \neq 0$ for some $u \in U$). Define $T: V \to W$ such that $\mathrm{T(v)} = \begin{cases} Sv \; if \; v \in U \\ 0 \; if \; v \in \mathrm{V} \; and \; v \notin U \end{cases}$. Prove that T is not a linear map on V
2. If $v_1, \ldots, v_m$ is a list of linearly independent vectors in $\mathbb{R}^n$, then for any $a \neq 0$ and index i, must $v_1, \ldots, v_{i-1}, av_i, \ldots, v_m$ be also linearly independent? Must they have the same span?
3. If $v_1, \ldots, v_m$ is a list of linearly independent vectors in $\mathbb{R}^n$, then for any $\mathrm{b} \neq 0$ and index $i \neq j$, must $v_1, \ldots, v_{i-1}, v_i + bv_j, \ldots, v_m$ be also linearly independent? Must they have the same span?
4. Find two linearly independent vectors in $\mathbb{R}^4$ on the plane $x + 2y - 3z - t = 0$. Then find three independent vectors? What about four?

($*$) marked problems are not as easy as others. Treat them as the most optional (optional-est?) thing among the whole course

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5. Suppose $v_1, \dots, v_6$ are six vectors in $\mathbb{R}^4$. These vectors:
   - (do)(do not)(might not) span $\mathbb{R}^4$
   - (are)(are not)(might be) linearly independent
   - Any 4 of those vectors (are)(are not)(might be) a basis for $\mathbb{R}^4$
6. Suppose S is a 5 dimensional subspace of $\mathbb{R}^6$. True or false (example if false):
   - Every basis for S can be extended to a basis for $\mathbb{R}^6$ by adding one more vector
   - Every basis for $\mathbb{R}^6$ can be reduced to a basis for S by removing one vector
7. ($*$)We can think of a permutation of n elements as a linear transformation $P: \mathbb{R}^n \to \mathbb{R}^n$ that permutes the basis elements $e_1, \dots, e_n$. In that case it is an $n \times n$ matrix with only 0's and 1's as entries and such that every row and every column have exactly one entry being 1. Is it true that for any n and any permutation P there exists an integer $k \geq 1$ such that $P^k = I$, where I is the identity matrix. Justify your answer.

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